cs/CpE 460 Introduction to robotics

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2-10 $R(y,\psi)\,R(x,\phi)\,R(z,\theta)$

$$\begin{bmatrix} \cos(\psi) & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\theta \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2-11 $R(z,\theta)\,R(x,\phi)\,R(x,\psi)$

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}$$

2-12 $R(z,\alpha)\,R(x,\phi)\,R(z,\theta)\,R(x,\psi)$

$$\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}$$

2-13 $R(z,\alpha)\,R(z,\theta)\,R(x,\phi)\,R(x,\psi)$

$$\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}$$

Tariq Alshugran

2-14

$$R(y,\tfrac{\pi}{2})\,R(x,\tfrac{\pi}{2}) = \begin{bmatrix} \cos\frac{\pi}{2} & 0 & \sin\frac{\pi}{2} \\ 0 & 1 & 0 \\ \sin\frac{\pi}{2} & 0 & \cos\frac{\pi}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ 0 & -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

2-15

$$\underline{R_1^2 = (R_2^1)^T} \quad \text{(make rows columns)}$$

$$\Rightarrow R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 1/2 \end{bmatrix}$$

$$\underline{R_c^b = R_a^b R_c^a}$$

$$\Rightarrow R_3^2 = R_1^2 R_3^1$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ \sqrt{3}/2 & 1/2 & 0 \\ 1/2 & -\sqrt{3}/2 & 0 \end{bmatrix}$$

Cs/CpE 460
Homework 2

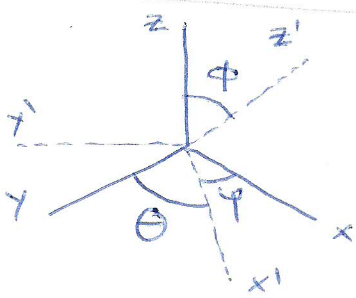
Tariq Alshugran

2-22 $R(x,\theta)\, R(y,\phi)\, R(z,\pi)\, R(y,-\phi)\, R(x,\theta)$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos(\phi) & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} \overset{(-1)}{\cos\pi} & \overset{(0)}{\sin\pi} & 0 \\ \overset{(0)}{\sin\pi} & \overset{(-1)}{\cos\pi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} -\cos(2\phi) & -2\cos\phi\sin\phi\sin\theta & \cos(\theta)\sin(2\phi) \\ -2\cos(\phi)\sin\phi\sin\theta & -(\cos\theta)^2 - \cos 2\phi(\sin\theta)^2 & -\cos(\phi)^2\sin(2\theta)\,\cancel{\sin} \\ \cos\theta\sin 2\phi & -\cos(\phi)^2\sin(2\theta) & \cos(\phi)^2\cos(\theta)^2 - \cos(\theta)^2\sin(\phi)^2 - \sin(\theta)^2 \end{bmatrix}$$

2-24 $\phi = \frac{\pi}{2}$, $\theta = 0$ and $\psi = \frac{\pi}{4}$



$$\begin{bmatrix} 0 & 0 & 1 \\ \cos\phi+\psi & -\sin\phi+\psi & 0 \\ \sin\phi+\psi & \cos\phi+\psi & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ \cos(3\pi/4) & -\sin(3\pi/4) & 0 \\ \sin(3\pi/4) & \cos(3\pi/4) & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

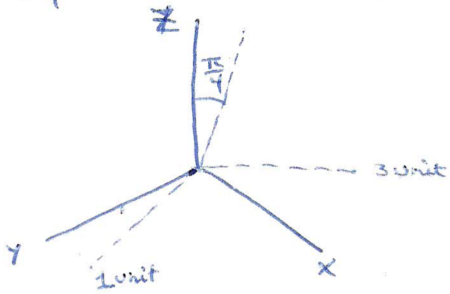
$\Rightarrow$ x direction is $\left(0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ $\begin{bmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

CS/CpE 460

Homework 2

Tariq Alshugran

2-37 Translation $(x, 3) \xrightarrow[\text{by}]{\text{Followed}}$ Rotation$(z, \frac{\pi}{2}) \xrightarrow[\text{by}]{\text{followed}}$ Translation $(y, 1)$



$$T(y,1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(x,3) = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(z, \tfrac{\pi}{2}) = \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} & 0 & 0 \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow$ $T = T(y,1)\, T(x,3)\, T(z, \frac{\pi}{2})$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

CS/CpE 460
Homework 2

Tariq Alshugran

* Table frame ($O_1 X_1 Y_1 Z_1$ to $O_0 X_0 Y_0 Z_0$)

$$H_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* Cube Frame ($O_2 X_2 Y_2 Z_2$ to $O_0 X_0 Y_0 Z_0$)

$$H_2^0 = \begin{bmatrix} 1 & 0 & 0 & (0.5-1) \\ 0 & 1 & 0 & (1+0.5) \\ 0 & 0 & 1 & (1+0.5-0.2-0.2) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* Camera frame ($O_3 X_3 Y_3 Z_3$ to $O_0 X_0 Y_0 Z_0$)

$$H_3^0 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* Cube Frame to Camera ($O_2 X_2 Y_2 Z_2$ to $O_3 X_3 Y_3 Z_3$)

$$H_2^3 = (H_3^0)^T H_2^0 = H_0^3 H_2^0$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -0.5 & 1.5 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$